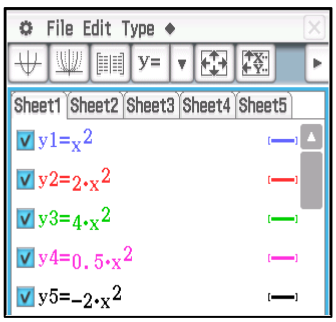
SADLER MATHEMATICS METHODS UNIT 1

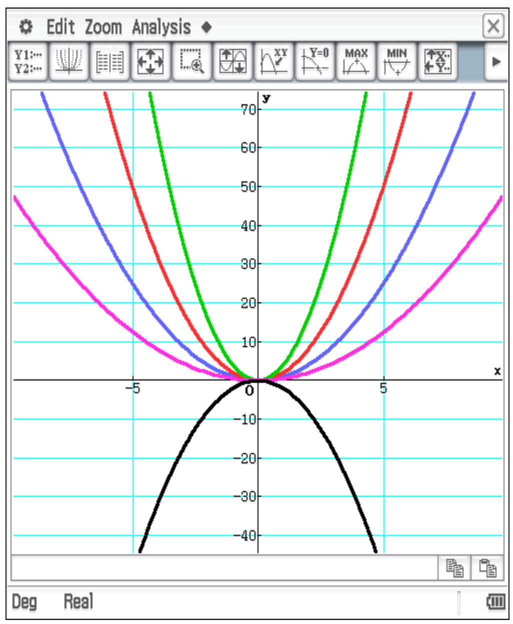
WORKED SOLUTIONS

Chapter 5 Quadratic functions

Exercise 5A

Question 1





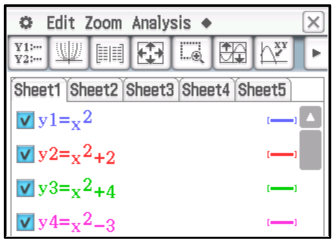
Changing the value of a in $y = ax^2$ effects the concavity of the graph and the rate of increase/decrease of the function.

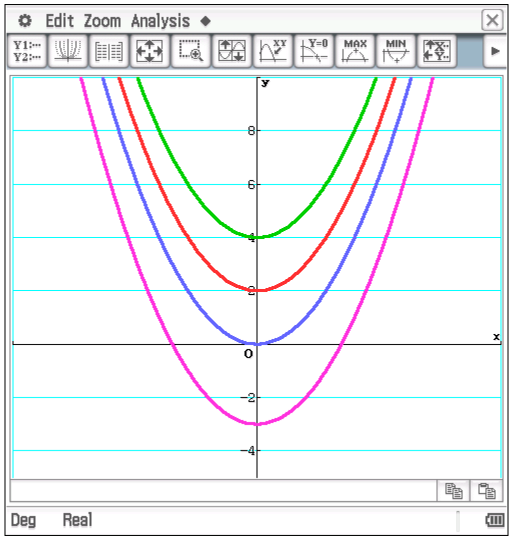
It transforms the basic parabola by stretching it in the *y*-direction by a factor of *a*.

When $a < 0$, our parabola becomes reflected in the *x*-axis or inverted.

The value of *a* effects the ‘steepness’ of the curve. For example the *y* values of each point on $y = 4x^2$ are four times the *y* values of each point of $y = x^2$.

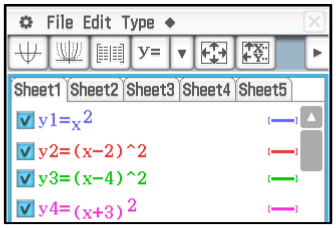
Question 2

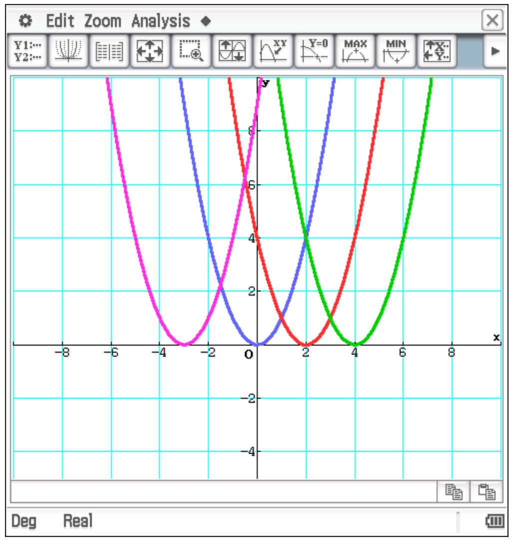




Changing the value of q in $y = x^2 + q$ changes the location of the y-intercept to (0, q). A positive value of q translates the basic parabola vertically q units up and a negative value of q translates the basic parabola vertically q units down.

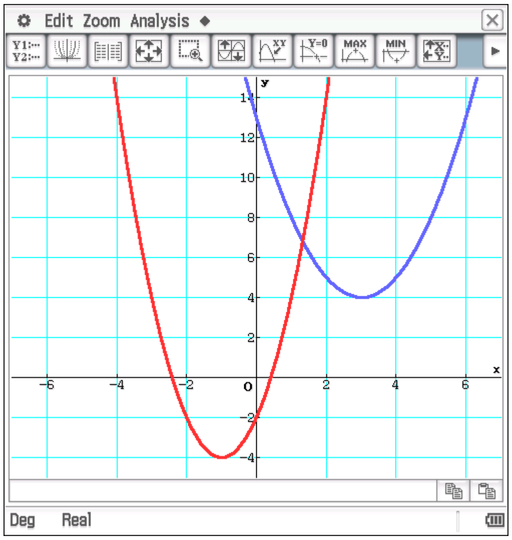
Question 3





Changing the value of p in $y=(x-p)^2$ changes the location of the x-intercept to (0, p). A positive value of p translates the basic parabola horizontally p units right and a negative value of p translates the basic parabola horizontally p units left.

Question 4



We would expect $y=(x-3)^2+4$ to have the same shape as $y=x^2$ (no stretching or change to the rate of change).

As $q=4$, we expect the parabola to be translated 4 units up.

As $p=3$, we expect the parabola to move 3 units right.

We would expect $y=2(x+1)^2-4$ to be steeper than $y=x^2$, as it have been stretched by a factor of 2 vertically.

As $q=-4$, we expect the parabola to be translated 4 units down.

As $p=-1$, we expect the parabola to move 1 units left.

Exercise 5B

Question 1

A has been translated 1 unit 'up': $y = x^2 + 1$
B has been translation 2 units down: $y = x^2 - 2$
C has been translated 4 units down: $y = x^2 - 4$
D has been translated 3 units right and 1 unit up: $y = (x-3)^2 + 1$
E has been translated 3 units left and 4 units down: $y = (x+3)^2 - 4$
F has been translated 2 units right and 3 units down: $y = (x-2)^2 - 3$

Question 2

G is $y = x^2$ inverted or reflected in x-axis: $\therefore y = -x^2$
H is G translated 3 units up: $y = -x^2 + 3$
I is G translated 3 units right: $y = -(x-3)^2$
J is G translated 3 units left and one unit up: $y = -(x+3)^2 + 1$

Question 3

K is $y = 2x^2$ translated 2 units down: $y = 2x^2 - 2$
L is $y = 2x^2$ translated 3 units right: $y = 2(x-3)^2$
M is $y = 2x^2$ translated 2 units left: $y = 2(x+2)^2$
N is $y = 2x^2$ translated 3 units right and 2 units down: $y = 2(x-3)^2 - 2$

Question 4

a $y = a(x-p)^2 + q$

$tp\ (-1,-4) \therefore\ p = 1, q = -4$

$y = a(x-1)^2 - 4$

Using y-intercept

$-1 = a(0-1)^2 - 4$

$-1 = a - 4$

$a = 3$

required equation : $y = 3(x-1)^2 - 4$

b $y = a(x-p)^2 + q$

$tp\ (3,8) \therefore\ p = 3, q = 8$

$y = a(x-3)^2 + 8$

Using y-intercept

$-10 = a(0-3)^2 + 8$

$-10 = 9a + 8$

$-18 = 9a$

$a = -2$

required equation : $y = -2(x-3)^2 + 8$

c $y = a(x-p)^2 + q$

$tp\ (4,-3) \therefore\ p = 4, q = -3$

$y = a(x-4)^2 - 3$

Using y-intercept

$5 = a(0-4)^2 - 3$

$5 = 16a - 3$

$8 = 16a$

$a = \frac{1}{2}$

required equation : $y = \frac{1}{2}(x-4)^2 - 3$

d $y = a(x - p)^2 + q$

$tp\ (-2, 10) \therefore p = -2, q = 10$

$y = a(x + 2)^2 + 10$

Using y-intercept

$$8 = a(0 + 2)^2 + 10$$

$$8 = 4a + 10$$

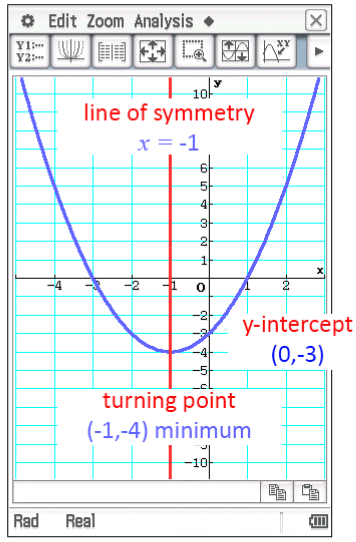
$$-2 = 4a$$

$$a = -\frac{1}{2}$$

required equation : $y = -\frac{1}{2}(x + 2)^2 + 10$

Exercise 5C

Question 1



a $x = -1$

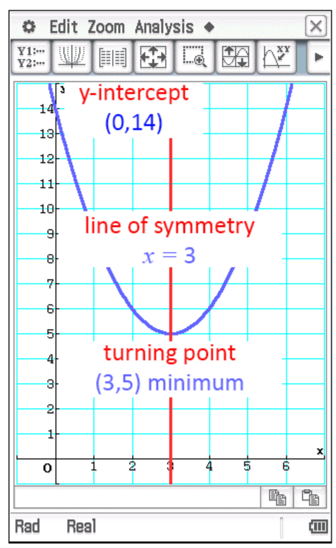
b tp $(-1, -4)$ min

c y-int $(0, ?)$

$y = (0+1)^2 - 4 = -3$

$(0, -3)$

Question 2



a $x = 3$

b tp $(3, 5)$ min

c y-int $(0, ?)$

$y = (0-3)^2 + 5 = -3$

$(0, 14)$

Question 3

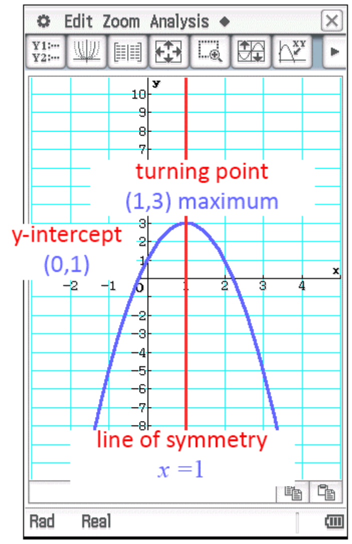
a $x = 1$

b tp $(1,3)$ max

c y-int $(0,?)$

$y = -2(0-1)^2 + 3 = 1$

$(0,1)$



Question 4

a y-int $(0,?)$

$y = (0-3)(0-7) = 21$

$(0,21)$

b x-int $(?,0)$

$y = (x-3)(x-7) = 0$

$\therefore x = 3, 7$

$(3,0)$ and $(7,0)$

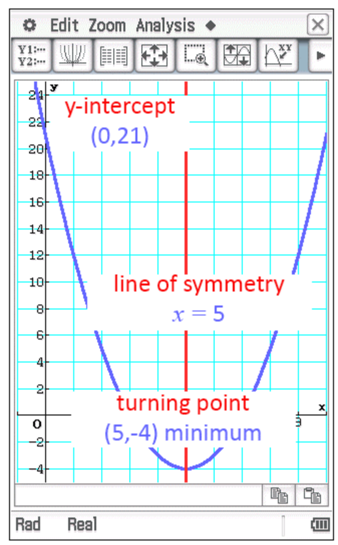
c line of symmetry

$$x = \frac{3+7}{2} = 5$$

d tp $(5,?)$

$y = (5-3)(5-7) = -4$

$(5,-4)$ min

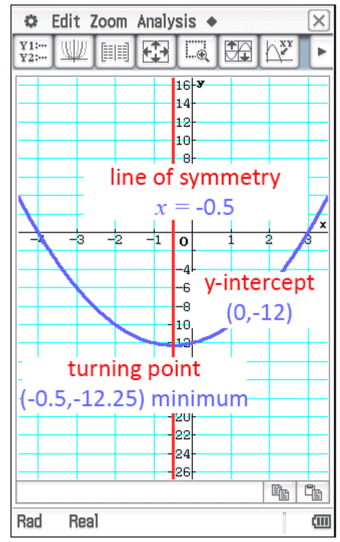


Question 5

a y-int $(0,?)$

$y=(0-3)(0+4)=-12$

$(0,-12)$



b x-int $(?,0)$

$y=(x-3)(x+4)=0$

$\therefore x=-4,3$

$(-4,0)$ and $(3,0)$

c line of symmetry

$$x=\frac{-4+3}{2}=-0.5$$

d tp (-0.5,?)

$y=(-0.5-3)(-0.5+4)=-12.25$

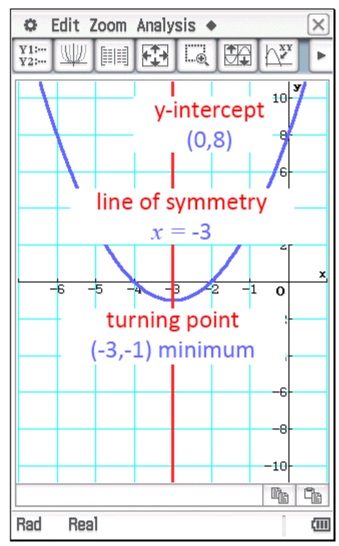
$(-0.5,-12.25)$ min

Question 6

a y-int $(0,?)$

$y=(0+2)(0+4)=8$

$(0,8)$



b x-int $(?,0)$

$y=(x+2)(x+4)=0$

$\therefore x=-4,-2$

$(-4,0)$ and $(-2,0)$

c line of symmetry

$$x=\frac{-4+(-2)}{2}=-3$$

d tp $(-3,?)$

$y=(-3+2)(-3+4)=-1$

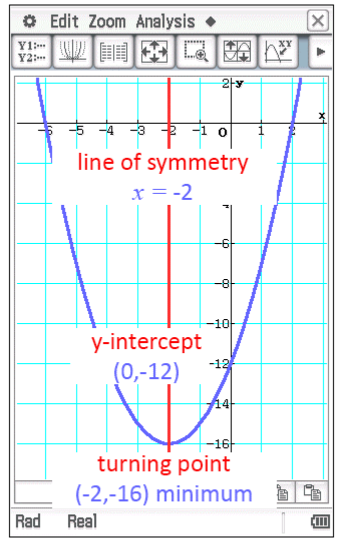
$(-3,-1)$ min

Question 7

a $x = -\frac{b}{2a}$

$= -\frac{4}{2}$

$= -2$



b $tp\ (-2, ?)$

$y = (-2)^2 + 4(-2) - 12 = -16$

$\therefore (-2, -16)$ min

c y-int $(0, ?)$

$y = 0^2 + 4(0) - 12 = -12$

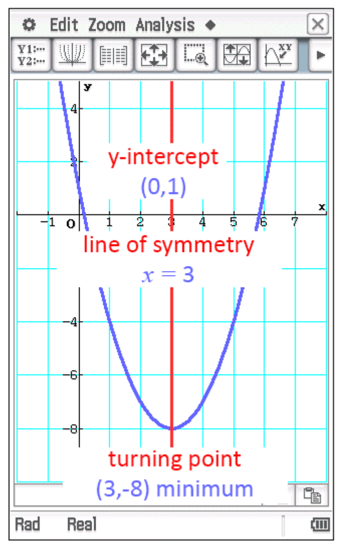
$\therefore\ (0, -12)$

Question 8

a $x = -\frac{b}{2a}$

$= \frac{6}{2}$

$= 3$



b $tp\ (3, ?)$

$y = (3)^2 - 6(3) + 1 = -8$

$\therefore (3, -8)$ min

c y-int $(0, ?)$

$y = 0^2 - 6(0) + 1 = 1$

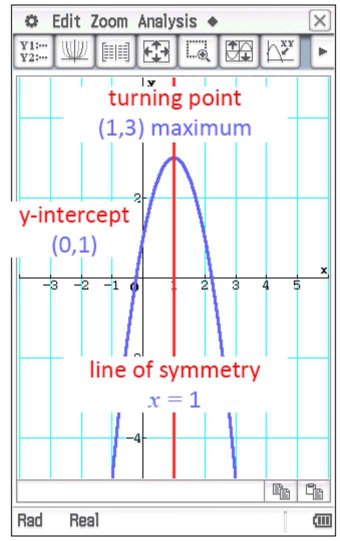
$\therefore\ (0, 1)$

Question 9

a $x = -\dfrac{b}{2a}$

$= \dfrac{-4}{2(-2)}$

$= 1$



b $tp\ (1,?)$

$y = -2(1)^2 + 4(1) + 1 = 3$

$\therefore (1,3)$ max

c $y\text{-int}\ (0,?)$

$y = -2(0)^2 + 4(0) + 1 = 1$

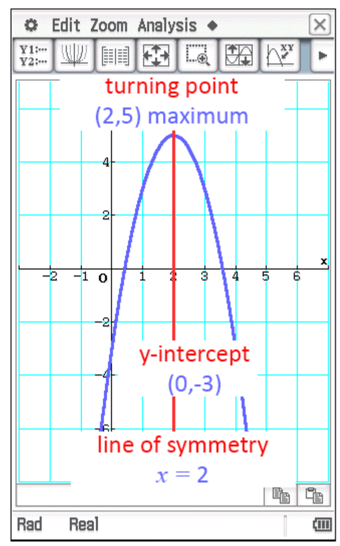
$\therefore\ (0,1)$

Question 10

a $x = -\dfrac{b}{2a}$

$= \dfrac{-8}{2(-2)}$

$= 2$



b $tp\ (2,?)$

$y = 8(2)^2 - 2(2) - 3 = 5$

$\therefore (2,5)$ max

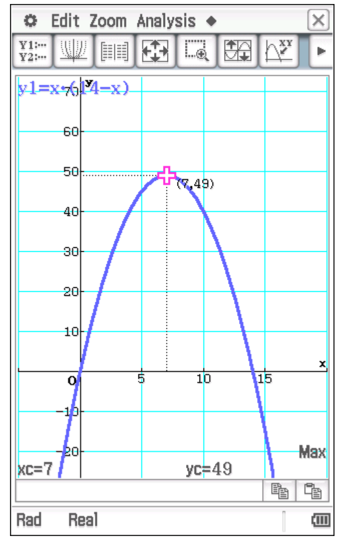
c $y\text{-int}\ (0,?)$

$y = 8(0)^2 - 2(0) - 3 = -3$

$\therefore\ (0,-3)$

Question 11

a There is 14m of fencing available to form the two unknown sides, x and y.



b $x+y=14$ (from part **a**)

$$y=14-x$$

$$A=l\times w$$
$$=x\times y$$
$$=x\times(14-x)$$

c See graph

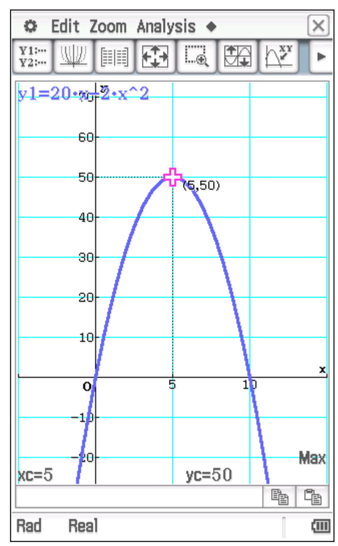
d Maximum area occurs at turning point.

Greatest area is 49 m^2 when $x=y=7\text{ m}$

Question 12

a We have three unknown sides to create from 20 m fencing i.e.

$$x+y+x=20$$
$$2x+y=20\Rightarrow y=20-2x$$



$$A=l\times w$$
$$=x\times(20-2x)$$
$$=20x-2x^2$$

b See graph

c Greatest area of $50\,\text{m}^2$ when $x=5\,\text{m}$ and $=20-2(5)=10\,\text{m}$

Question 13

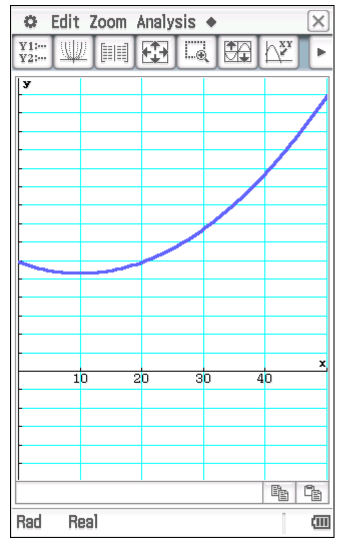
a A is the turning point of the curve: (2.5, 11.25)

b h is the y-intercept

$$h = -0.2(0-2.5)^2 + 11.25$$
$$= 10$$

c The graph is concave down.

Question 14



a $P = 0.6(0)^2 - 12(0) + 590$
$= 590$

The average house price is $590 000.

b $P = 0.6(15)^2 - 12(15) + 590$
$= 545$

The average house price is $545 000.

c When buying a house, you want the purchase price

to be a minimum.

Using classpad, the turning point of the Price equation is (10, 530)

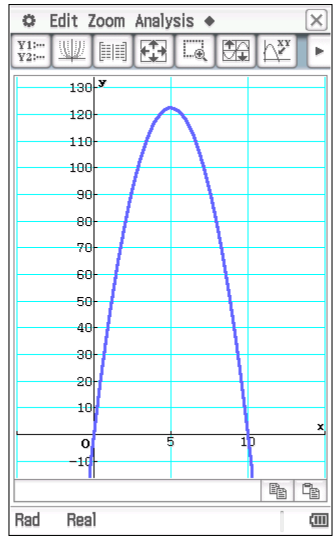
indicating the lowest price would be $530 000 after 10 months.

Question 15

Graph will have a maximum turning point due to negative co-efficient of t^2

Co-ordinates of turning point are (5, 122.5)

Maximum height of 122.5 m achieved when $t = 5$ seconds



Question 16

a Section AB is concave up.

b C is the turning point of $y = \frac{3}{160}(x-40)^2 + 15$

Coordinates of turning point (40, 15) so the bridge is 15 m above the water level

c Line of symmetry passes through turning point, $x = 40$

d D is on the y-axis, $x = 0 \therefore \text{DC} = 40\text{ m}$

e $\text{DE} = 2 \times 40 = 80\text{ m}$ due to symmetry

f A is the y-intercept, $x = 0$.

$$\text{A} = \frac{3}{160}(0-40)^2 + 15 = 45\text{ m}$$

As D is 15 m above the water level, DA = 30 m.

Question 17

a Both arches are concave down.

b The midpoint is halfway between the *x*-intercepts of supporting arch

or the *x* co-ordinate of turning point. Using classpad, $x = 150\text{ m}$.

c One quarter along the bridge is 75 m.

Length of vertical strut required is the difference in *y*-values of the two arches when *x* = 75.

$$\text{Road arch}: y = -\frac{75^2}{2250} + \frac{2 \times 75}{15} + 40 = 47.5$$

$$\text{Supporting arch}: y = \frac{2 \times 75}{3} - \frac{75^2}{450} = 37.5$$

$$47.5\text{ m} - 37.5\text{m} = 10\text{ m}$$

$\therefore$ strut is 10 m in length

d The x-axis is 4 m higher than the low tide and 4 m lower than high tide.

Coordinates of turning point, by classpad, (150, 50).

Bridge is 50 m above mean water level at its highest point and therefore

i at low tide, it is 54 m clear of the water

ii at high tide it is 46 m clear of the water

Exercise 5D

Question 1

x	0	1	2	3	4	5
y	5	12	21	32	45	60

7	9	11	13	15

2	2	2	2

Relationship is quadratic with a constant second difference of 2, $\Rightarrow a = 1$

When $x = 0, y = 5 \Rightarrow c = 5$

We have $y = x^2 + bx + 5$

When $x = 1, y = 12$

$12 = 1 + b + 5$

$b = 6$

The required equation is $y = x^2 + 6x + 5$.

Question 2

x	0	1	2	3	4	5
y	0	1	8	27	64	125

1	7	19	37	61

6	12	18	24

Reader should identify cubic numbers by inspection but first and second differences are not constant therefore the relationship is not linear or quadratic

Question 3

x	0	1	2	3	4	5
y	3	5	9	15	23	33
	2	4	6	8	10	
		2	2	2	2	

Relationship is quadratic with a constant second difference of 2, $\Rightarrow a = 1$

When $x = 0, y = 3 \Rightarrow c = 3$

We have $y = x^2 + bx + 3$
When $x = 1, y = 5$
$5 = 1 + b + 3$
$b = 1$
The required equation is $y = x^2 + x + 3$.

Question 4

x	0	1	2	3	4	5
y	1	6	11	16	21	26
	5	5	5	5	5	

A constant first difference indicates the relationship is linear.

A difference of 5 indicates the gradient is 5.

We have $y = 5x + c$
When $x = 0, y = 1 \Rightarrow c = 1$
The required equation is $y = 5x + 1$.

Question 5

x	0	1	2	3	4	5
y	2	3	6	11	18	27

1	3	5	7	9

2	2	2	2

Relationship is quadratic with a constant second difference of 2, $\Rightarrow a = 1$

When $x = 0, y = 2 \Rightarrow c = 2$

We have $y = x^2 + bx + 2$
When $x = 1, y = 3$
$3 = 1 + b + 2$
$b = 0$
The required equation is $y = x^2 + 2$.

Question 6

x	0	1	2	3	4	5
y	π	2π	3π	4π	5π	6π

π	π	π	π	π

A constant first difference indicates the relationship is linear.

A difference of π indicates the gradient is π.

We have $y = \pi x + c$
When $x = 0, y = \pi \Rightarrow c = \pi$
The required equation is $y = \pi x + \pi$.

Question 7

x	0	1	2	3	4	5
y	3	6	12	24	48	96
	3	6	12	24	48	
		3	6	12	24	

Reader should identify numbers are doubling (indicating an exponential relationship) by inspection but first and second differences are not constant therefore the relationship is not linear or quadratic

Question 8

x	0	1	2	3	4	5
y	4	10	18	28	40	54
	6	8	10	12	14	
		2	2	2	2	

Relationship is quadratic with a constant second difference of 2, $\Rightarrow a = 1$

When $x = 0, y = 4 \Rightarrow c = 4$

We have $y = x^2 + bx + 4$

When $x = 1, y = 10$

$10 = 1 + b + 4$

$b = 5$

The required equation is $y = x^2 + 5x + 4$.

Question 9

x	0	1	2	3	4	5
y	3	11	19	27	35	43
	8	8	8	8	8	

A constant first difference indicates the relationship is linear.

A difference of 8 indicates the gradient is 8.

We have $y = 8x + c$

When $x = 0, y = 3 \Rightarrow c = 3$

The required equation is $y = 8x + 3$.

Question 10

x	0	1	2	3	4	5
y	3	5	11	21	35	53
	2	6	10	14	18	
		4	4	4	4	

Relationship is quadratic with a constant second difference of 4, $\Rightarrow a = 2$

When $x = 0, y = 3 \Rightarrow c = 3$

We have $y = 2x^2 + bx + 3$

When $x = 1, y = 5$

$5 = 2(1) + b + 3$

$b = 0$

The required equation is $y = 2x^2 + 3$.

Question 11

x	0	1	2	3	4	5
y	13	4	1	4	13	28
		–9	–3	3	9	15
			6	6	6	6

Relationship is quadratic with a constant second difference of 6, $\Rightarrow a = 3$

When $x = 0, y = 13 \Rightarrow c = 13$

We have $y = 3x^2 + bx + 13$

When $x = 1, y = 4$

$4 = 3(1) + b + 13$

$b = -12$

The required equation is $y = 3x^2 - 12x + 13.$

Question 12

x	–2	0	2	4	6	8
y	–20	–4	4	4	–4	–20
	16	8	0	–8	–16	
		–8	–8	–8	–8	

Relationship is quadratic with a constant second difference of –8, however the spacing of the *x* values means we cannot directly determine the value of a

When $x=0, y=-4 \Rightarrow c=-4$

We have $y=ax^2+bx-4$

When $x=2, y=4$

$4=a(2)^2+2b-4$

$8=4a+2b \qquad \Rightarrow 2b=8-4a$

When $x=4, y=4$

$4=a(4)^2+42b-4$

$8=16a+4b$

Solving simultaneously,

$8=16a+2(8-4a)$

$8=8a+16$

$a=-1$

$b=6$

The required equation is $y=-x^2+6x-4$.

Question 13

a

L	1	2	3	4	5	6
n	6	24	54	96	150	216
	18	30	42	54	66	
		12	12	12	12	

b Relationship is quadratic with a constant second difference of 124, $\Rightarrow a = 6$

c $n = 6L^2$

Question 14

a

r	1	2	3	4	5	6
n	1	3	6	10	15	21
	2	3	4	5	6	
		1	1	1	1	

b Relationship is quadratic with a constant second difference of 2, $\Rightarrow a = 0.5$

c By extending the table, we find when $r = 0, n = 0 \Rightarrow c = 0$

We have $n = 0.5r^2 + br$

When $r = 1, n = 1$

$1 = 0.5 + b$

$b = 0.5$

The required equation is $n = 0.5r^2 + 0.5r$

$= 0.5r(r+1)$

Exercise 5E

Question 1

$$\begin{aligned} y &= (x+2)^2 - 4 - 1 \\ &= (x+2)^2 - 5 \end{aligned}$$

Minimum turning point at $(-2,-5)$.

Question 2

$$\begin{aligned} y &= (x-3)^2 - 9 + 2 \\ &= (x-3)^2 - 7 \end{aligned}$$

Minimum turning point at $(3,-7)$.

Question 3

$$\begin{aligned} y &= (x-4)^2 - 16 + 10 \\ &= (x-4)^2 - 6 \end{aligned}$$

Minimum turning point at $(4,-6)$.

Question 4

$$\begin{aligned} y &= (x+3)^2 - 9 + 3 \\ &= (x+3)^2 - 6 \end{aligned}$$

Minimum turning point at $(-3,-6)$.

Question 5

$$\begin{aligned} y &= (x-1.5)^2 - 2.25 + 2 \\ &= (x-1.5)^2 - 0.25 \end{aligned}$$

Minimum turning point at $(1.5,-0.25)$.

Question 6

$$\begin{aligned} y &= (x-2.5)^2 - 6.25 + 3 \\ &= (x-2.5)^2 - 3.25 \end{aligned}$$

Minimum turning point at $(2.5, -3.25)$.

Question 7

$$\begin{aligned} y &= -(x^2 - 10x + 1) \\ &= -[(x-5)^2 - 25 + 1] \\ &= -[(x-5)^2 - 24] \\ &= -(x-5)^2 + 24 \end{aligned}$$

Maximum turning point at $(5, 24)$.

Question 8

$$\begin{aligned} y &= 2(x^2 - 6x + 1.5) \\ &= 2[(x-3)^2 - 9 + 1.5] \\ &= 2[(x-3)^2 - 7.5] \\ &= 2(x-3)^2 - 15 \end{aligned}$$

Minimum turning point at $(3, -15)$.

Question 9

$$\begin{aligned} y &= -2(x^2 - 4x - 2) \\ &= -2[(x-2)^2 - 4 - 2] \\ &= -2[(x-2)^2 - 6] \\ &= -2(x-2)^2 + 12 \end{aligned}$$

Maximum turning point at $(2, 12)$.

Question 10

$$\begin{aligned}y &= 2(x^2 + 2.5x + 2)\\ &= 2[(x+1.25)^2 - 1.5625 + 2]\\ &= 2[(x+1.25)^2 + 0.4375]\\ &= 2(x+1.25)^2 + 0.875\end{aligned}$$

Minimum turning point at $(-1.25, 0.875)$.

Miscellaneous exercise five

Question 1

a $f(6)=5(6)+1=31$

b $g(2)=2^2-3=1$

c
$$\begin{aligned}&f(2)+g(6)\\&=5(2)+1+6^2-3\\&=44\end{aligned}$$

Question 2

a concave down

b concave up

c concave down

Question 3

$y=2x-5$

A $a=2(5)-3=1$

B $b=2(2)-5=-1$

C $c=2(-4)-5=-13$

D $d=2(2.5)-5=0$

E $13=2e-5$

$18=2e$

$e=9$

F $-5=2f-5$

$0=2f$

$f=0$

Question 4

a $2 \times m = -1 \Rightarrow m = -\frac{1}{2}$

b $y = 3x - 4$ has $m = 3$

$3 \times m = -1 \Rightarrow m = -\frac{1}{3}$

c $y = -0.2x + 1$ has $m = -0.2$

$-0.2 \times m = -1 \Rightarrow m = 5$

d $y = -0.5x + 1$ has $m = -0.5$

$-0.5 \times m = -1 \Rightarrow m = 2$

$y = 2x + c$

$13 = 2(3) + c$

$c = 7$

required equation : $y = 2x + 7$

Question 5

a y-intercept, $x = 0$

$y = (0-1)(0-3) = 3$

$\therefore (0, 3)$

b x-intercepts, $y = 0$

$0 = (x-1)(x-3)$

$x = 1, 3$

$\therefore (1, 0)$ and $(3, 0)$

c line of symmetry

$x = \frac{1+3}{2} = 2$

$\therefore x = 2$

d turning point (2, ?)

$y = (2-1)(2-3) = -1$

$\therefore$ minimum tp $(2, -1)$

Question 6

Equation	y-axis intercept	Line of symmetry	Turning point coordinates	Max or Min
$y = x^2 + 4x + 1$	$(0,1)$	$x = -\frac{4}{2} = -2$	$y = (-2)^2 + 4(-2) + 1 = -3$ $\therefore (-2, -3)$	minimum
$y = x^2 - 2x - 1$	$(0,-1)$	$x = -\frac{(-2)}{2} = 1$	$y = (1)^2 - 2(1) - 1 = -2$ $\therefore (1, -2)$	minimum
$y = 2x^2 + 4x - 3$	$(0,-3)$	$x = -\frac{4}{4} = -1$	$y = 2(-1)^2 + 4(-1) - 3 = -5$ $\therefore (-1, -5)$	minimum
$y = 2x^2 + 6x - 1$	$(0,-1)$	$x = -\frac{6}{4} = -1.5$	$y = 2(-1.5)^2 + 6(-1.5) - 1 = -5.5$ $\therefore (-1.5, -5.5)$	minimum

Question 7

a line of symmetry : $x = -3$

b turning point $(-3, -4)$

c A move of two units right moves the line of symmetry two units right : $x = -1$

d The turning point is affected by both moves: $(-1, -1)$

Question 8

A	$x = 4$		
B	$y = -3$		
C	$y - \text{int}\,(0,0)$	$m = 1$	$y = x$
D	$y\text{-int}\,(0,2)$	$m = 1$	$y = x + 2$
E	$y\text{-int}\,(0,4)$	$m = 2$	$y = 2x + 4$
F	$y - \text{int}\,(0,0)$	$m = -1$	$y = -x$
G	$y\text{-int}\,(0,4)$	$m = \frac{1}{4}$	$y = 0.25x + 4$
H	$y\text{-int}\,(0,1)$	$m = \frac{1}{2}$	$y = 0.5x + 1$
I	$y\text{-int}\,(0,-1)$	$m = -\frac{1}{2}$	$y = -0.5x - 1$

Question 9

Curve I: minimum turning point and x-intercepts of $(1,0)$ and $(3,0)$ $\Rightarrow y = (x-1)(x-3)$

Curve II: maximum turning point and x-intercepts of $(-2,0)$ and $(2,0) \Rightarrow y = -(x-2)(x+2)$

Curve III: maximum turning point and x-intercepts of $(-3,0)$ and $(-1,0) \Rightarrow y = -(x+1)(x+3)$

Curve IV: minimum turning point and x-intercepts of $(-3,0)$ and $(-1,0) \Rightarrow y = (x+1)(x+3)$

Question 10

a

x	1	2	3	4	5	6	7	8
y	7	10	13	16	19	22	25	28

A constant first difference of 3 means the relationship is of the form $y = 3x + c$
Using (5,19) from the table,

$19 = 3(5) + c \Rightarrow c = 4$
$\therefore$ required rule is $y = 3x + 4$.

b

x	1	2	3	4	5	6	7	8
y	1	3	5	7	9	11	13	15

A constant first difference of 2 means the relationship is of the form $y = 2x + c$

Using (7,13) from the table,

$13 = 2(7) + c \Rightarrow c = -1$
$\therefore$ required rule is $y = 2x - 1$.

c

x	1	2	3	4	5	6	7	8
y	15	13	11	9	7	5	3	1

A constant first difference decreasing by 2 means the relationship is of the form $y = -2x + c$

Using (3,11) from the table,

$11 = -2(3) + c \Rightarrow c = 17$
$\therefore$ required rule is $y = -2x + 17$.

d

x	1	2	3	4	5	6	7	8
y	4	9	14	19	24	19	32	39

$$\frac{19-9}{2}=5$$

A constant first difference of 5 means the relationship is of the form $y=5x+c$

Using (2,9) from the table,

$9=5(2)+c \Rightarrow c=-1$

$\therefore$ required rule is $y=5x-1$.

e

x	1	2	3	4	5	6	7	8
y	1				13			

$$\frac{13-1}{5-1}=\frac{12}{4}=3$$

A constant first difference of 3 means the relationship is of the form $y=3x+c$

Using (5,13) from the table,

$13=3(5)+c \Rightarrow c=-2$

$\therefore$ required rule is $y=3x-2$.

Question 11

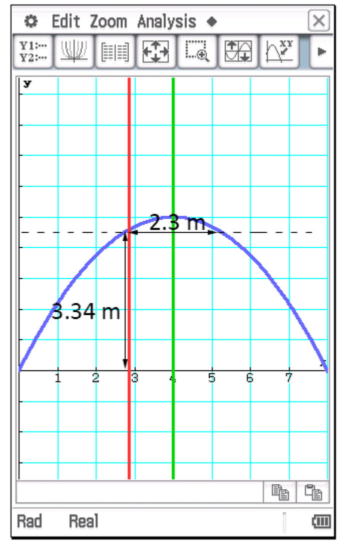
$y=3(x-2)^2+c$

Using (0,15),

$15=3(0-2)^2+c \Rightarrow c=3$

$\therefore$ required equation is $y=3(x-2)^2+3$.

Question 12



a The road width would be the distance between the *x*-intercepts.

x-intercepts, $y = 0$

$$0 = \frac{5x}{16}(8 - x)$$

$$x = 0, 8$$

The road is 8m wide.

b The clearance at the centre is the height of the turning point.

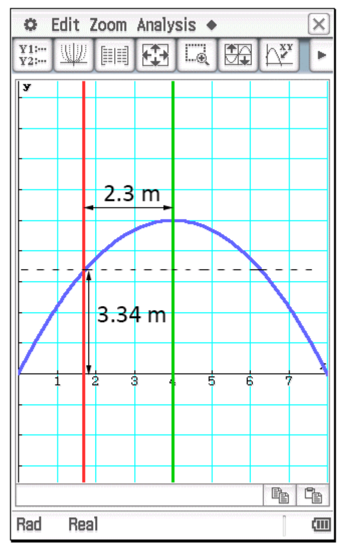
By classpad, turning point is (4, 5).

The clearance at the centre is 5 m.

c $4 - 2.3 = 1.7$

$$y = \frac{5(1.7)(8 - 1.7)}{16} = 3.346875$$

$\therefore$ the truck can be a maximum height of 3.34 m.



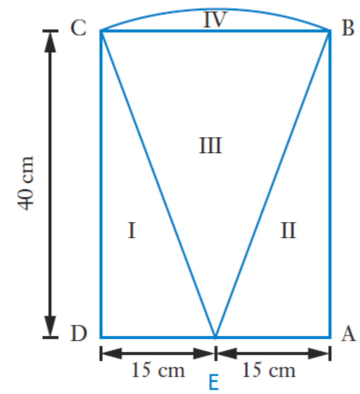
d $4 - \frac{2.3}{2} = 2.85$

$$y = \frac{5(2.85)(8 - 2.85)}{16} = 4.58671875$$

$\therefore$ the truck can be a maximum height of 4.58 m.

Question 13

a Area of triangular pieces I and II :



$0.5 \times 15 \times 40 = 300\,\text{cm}^2$

By symmetry, piece III has the same area as I and II combined

$600\,\text{cm}^2$

$CE^2 = 40^2 + 15^2$
$CE = 42.7\,\text{cm}$

We know the area of piece III and using the area of a triangle rule we can find $\angle CEA$.

$$600 = 0.5 \times 42.7^2 \times \sin \angle CEA$$
$$\angle CEA = 0.72$$

Area of piece IV

$$0.5 \times 42.72^2 (0.72 - \sin(0.72))$$
$$= 55\,\text{cm}^2$$

(Other methods may produce slightly different answers)

b The only length not known or previous calculated is the arc AB.

$$AB = 42.7 \times 0.72$$
$$= 30.0\,\text{cm}$$

Total length of lead :

$(40 + 30 + 42.7) \times 2 + 30.2 = 255.6\,\text{cm}$
256 cm of lead required